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EFFECT OF SKEW ANGLE ON RIGID-FRAME REACTIONS

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STRUCTURAL DIVISION

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PAPERS

EFFECT OF SKEW ANGLE ON
RIGID-FRAME REACTIONSBY WALTER C. BOYER,¹ JUN. ASCE

SYNOPSIS

Modern high-speed traffic requires grade-separation structures that conform to essential road alinement. The two-hinged, rigid-frame bridge is highly regarded for such use; but, because of past failures, some doubt exists as to the applicability of the skewed type of structure. The investigation on which this paper is based was made to confirm the validity of the analytical method of solution for reactive forces in a skewed structure with models. The variation of these forces was studied for a complete range of skew angles.

INTRODUCTION

The problem of the skew rigid frame has always held a prominent place in engineering study and research. It assumed notoriety in its initial stages when several failures, entailing loss of life, resulted from fallacious design procedure. Following this period of skew span failures, the designing engineer resorted to another design which became perpetuated in structures that exist today as monuments to his inability to cope with the skew bridge problem—that is, the sharp-bend bridge, which reduced the design to a right frame but which created a traffic hazard of the first magnitude.

The profession can view with pride the first complete and logical mathematical solution of the skewed span problem presented by J. Charles Rathbun,² M. ASCE, in 1924. In this analysis Mr. Rathbun correctly included the effects of torsion on the redundant reactions and the moments inherent in a skewed arch. The validity of his solution was questioned and consequently tests were conducted by the late George E. Beggs,³ M. ASCE. The series of tests were

NOTE.—Written comments are invited for publication; the last discussion should be submitted by March 1, 1951.

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² "Analysis of the Stresses in the Ring of a Concrete Skew Arch," by J. Charles Rathbun, *Transactions, ASCE*, Vol. LXXXVIII, 1924, p. 611.

³ "Skew Arch Reactions Measured by Reciprocal Method," by George E. Beggs, *Engineering News-Record*, July 21, 1927, pp. 106-107.

made for a 30° angle of skew and for varying roadway widths. Since the publication of this work, doubt has still been raised concerning the validity of the mathematical solution when the skew angle becomes large and the slab wide. Thus, it is evident additional experimental research is required as a comparison to the analytical solution.

STATEMENT OF THE PROBLEM

This paper is concerned primarily with testing the validity of Mr. Rathbun's theory when applied to a two-hinged skewed rigid structure, and with showing the variation of the various reactive moments and forces with the angle of skew. The basic structure used had a slab 100 ft long with a leg height of 22 ft. This structure, when scaled down, was ideal for model study. It was tested for the effects of skew by varying the angle from 0° to 50° , in increments of 10° .

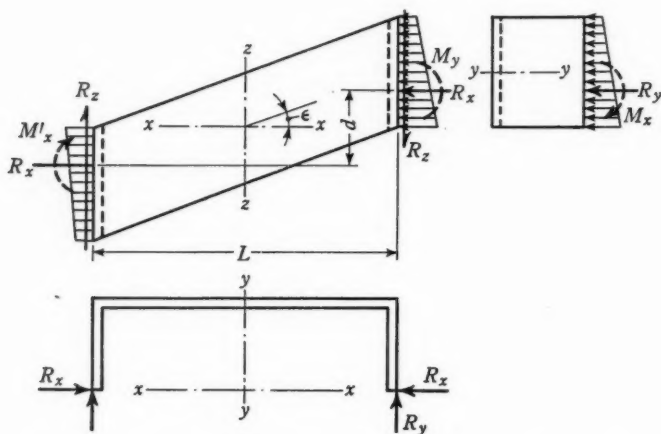


FIG. 1.—REACTIVE FORCES AND MOMENTS IN A TWO-HINGED SKEWED FRAME

The roadway slab width was varied from 20 ft to 40 ft in increments of 10 ft. The solution by these data was compared to the solution of the reactions of the structure by mathematical analyses.

The mathematical design procedure used was basically that proposed by Mr. Rathbun and applied by Arthur G. Hayden,⁴ M. ASCE, to the two-hinged rigid frame. The nomenclature used for reactive forces is depicted in Fig. 1.

LABORATORY SETUP

The fundamental method used for determining the reactive elements was based on the deformer method presented by Mr. Beggs.⁵ This system has the advantage of yielding influence lines for reactions and moments by introducing controlled deformations at the reactions. In contrast to the pro-

⁴ "The Rigid Frame Bridge," by Arthur G. Hayden, John Wiley & Sons, Inc., New York, N. Y., 1941, pp. 137-182.

⁵ "The Use of Models in the Solution of Indeterminate Structures," by George E. Beggs, *Journal of the Franklin Institute*, March, 1927, p. 375.

cedure, followed by Mr. Beggs, of introducing extremely small deflections, the jig developed for this series of tests employed relatively larger deflections as prescribed in the simplification of William J. Eney,⁶ M. ASCE. A diagram of the movable abutment through which controlled deformations and moments were introduced into the structure is shown in Fig. 2. A thrust along axis z to determine R_z may be applied by sliding the base plates along angle slides A, and maintaining this deformation by the gage blocks behind butt angle B. A thrust along axis x to determine R_x may be applied by sliding the channel section along the normal blocks, guided by the moment shaft. The deformation is controlled by placing gage blocks behind butt stop C. Moment M_x is

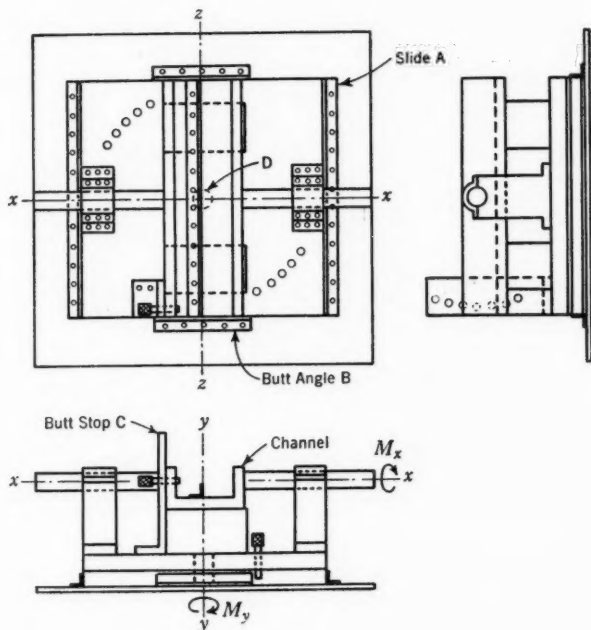


FIG. 2.—DIAGRAM OF TEST STAND

applied by removing the normal blocks and rotating the channel section. The angle of twist is adjusted by the calibrated radial pitchholes in butt stop C. A pin is used to maintain this deformation. Moment M_y is applied by turning one base plate over the other about the moment fulcrum D. The angle is adjusted by the fixed radial pitch in the upper base plate and is held by a pin.

A converted hydraulic point gage, accurate to ± 0.003 in. was used to measure vertical deflections along the slab of the model. To bring the point gage into contact with the model slab without deforming the surface, a conducting silver paste was applied to the surface of the model. A "magic eye"

⁶"Stress Analysis with Elastic Models and a New Deformeter Apparatus," by William J. Eney, Report No. 214.19, Lehigh Univ., Bethlehem, Pa., 1941.

tube (6E5) was used to determine the exact contact of the point gage with the silvered slab surface.

The models were constructed to a scale of 1 in. = 5 ft, thus giving a span of 20 in. for a prototype of 100 ft. The ratio of the moment of inertia of the slabs to the moment of inertia of the legs was 1:2. The slab was connected to the legs by steel clamps making a rigid joint. Hinged ends were contrived by adopting a bank of cabinet hinges, carefully selected, to reduce friction to a minimum.

Several materials were tested for model construction, and grade XXX phenolic resin was ultimately selected since it exhibited a minimum of cold flow characteristics.

Fig. 3 shows the grid system used to give complete deflection coverage of the slab. After a controlled movement had been introduced by the test frame, vertical deflection readings were taken at each of the grid points by the point

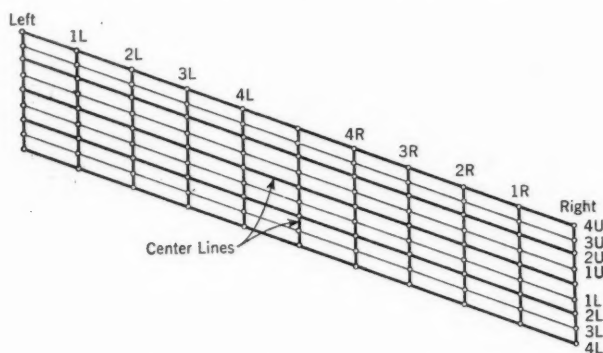


FIG. 3.—PLAN OF A SKEWED FRAME SHOWING THE GRID SYSTEM FOR DEFLECTION READINGS

gage mounted on a fixed and independent datum plane. The datum plane used was a 3-ft by 3-ft steel surface plate, 1.5 in. thick, accurate to 0.001 in. A photograph of the test apparatus in normal position is shown in Fig. 4.

EXAMPLE OF A 20-FT ROADWAY WITH A SKEW ANGLE OF 30°

To determine the thrust along R_x , the test stand was given a deflection of 0.5 in. along the x -axis from the normal position, this deflection being introduced in both directions. Vertical deflection readings were taken at all grid points along the surface from the normal position in both directions. Thus, the difference in readings represented the vertical deflections of the slab for an abutment deflection of 1 in. The readings taken for the center line of the model were as recorded in Cols. 2 and 3, Table 1.

Since the point gage reads in centimeters the difference values (Col. 4, Table 1) must be divided by 2.54 to convert them to influence ordinates. The influence ordinates along other longitudinal sections have the same values as those indicated for the center line. The influence ordinates for R_x were found in the same manner as those for R_z .

To determine the moment M_x , the channel was turned first in a clockwise direction. Vertical deflection readings were taken at all grid points along the surface of the model slab. The channel was then turned in a counterclockwise

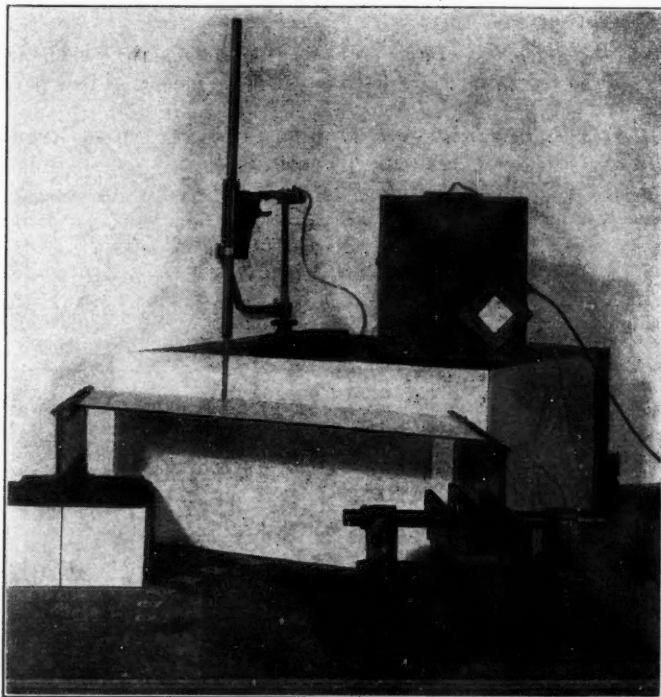


FIG. 4.—TEST APPARATUS IN NORMAL POSITION

direction and readings were taken again. The readings thus taken along line 4U, Fig. 3, were as shown in Cols. 5 and 6, Table 1.

TABLE 1.—TESTS IN A MODEL OF A 30° SKEW BRIDGE, 20 FT WIDE

Road section (Fig. 3)	THRUST ALONG R_x			MOMENT, M_x			Influence ordinate (line 2, Fig. 3)
	Movement left	Movement right	Difference	Clockwise	Counterclockwise	Difference	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Left end.....	19.55	19.55	0	19.56	19.55	0.01	0
1L.....	19.17	18.54	0.63	19.16	19.09	0.07	- 0.64
2L.....	19.09	18.07	1.02	19.15	18.90	0.25	- 2.25
3L.....	19.05	17.73	1.32	19.15	18.75	0.40	- 3.67
4L.....	19.01	17.52	1.49	19.23	18.61	0.62	- 5.68
Center.....	19.01	17.46	1.55	19.27	18.51	0.76	- 6.96
4R.....	19.03	17.54	1.49	19.34	18.46	0.88	- 8.07
3R.....	19.06	17.75	1.31	19.45	18.45	1.00	- 9.16
2R.....	19.11	18.09	1.02	19.57	18.46	1.11	-10.20
1R.....	19.14	18.50	0.68	19.67	18.47	1.20	-11.00
Right end....	19.46	19.46	0	20.00	18.74	1.26	-11.52

To convert the difference values to inches, they must be divided by 2.54. The influence ordinates for moment were then determined by the relationship:

$$M_x = 1 \times \frac{y_1}{y} \times l \times n \dots \dots \dots (1)$$

in which y_1 is the deflection of any point on the model slab; y is the introduced deflection; l is the moment arm; and n is the scale factor, in feet per inch.

TABLE 2.—COMPARISON OF MODEL AND ANALYTICAL RESULTS
FOR THE REACTIONS R

(These Results Are the Same for 20-Ft, 30-Ft and 40-Ft Roadways)

Skew angle	INFLUENCE POINTS ALONG CENTERLINE OF SLAB										
	Left	1L	2L	3L	4L	Center	4R	3R	2R	1R	Right
(a) MODEL RESULTS, R_x											
0	0	0.228	0.394	0.506	0.580	0.612	0.580	0.506	0.394	0.228	0
10	0	0.209	0.330	0.505	0.577	0.612	0.577	0.505	0.330	0.209	0
20	0	0.228	0.394	0.506	0.571	0.612	0.577	0.506	0.394	0.221	0
30	0	0.247	0.402	0.520	0.588	0.610	0.588	0.518	0.402	0.252	0
30	0	0.236	0.406	0.510	0.586	0.615	0.590	0.524	0.414	0.242	0
50	0	0.248	0.430	0.545	0.579	0.618	0.579	0.545	0.430	0.248	0
(b) ANALYTICAL RESULTS, R_x											
0	0	0.224	0.399	0.523	0.598	0.623	0.598	0.523	0.399	0.224	0
10	0	0.224	0.399	0.528	0.598	0.621	0.598	0.528	0.399	0.224	0
20	0	0.224	0.400	0.524	0.598	0.622	0.598	0.524	0.400	0.224	0
30	0	0.224	0.399	0.523	0.598	0.622	0.598	0.523	0.399	0.224	0
40	0	0.224	0.400	0.523	0.598	0.623	0.598	0.523	0.400	0.224	0
50	0	0.224	0.398	0.523	0.598	0.623	0.598	0.523	0.398	0.224	0
(c) MODEL RESULTS, R_z											
0	0	0	0	0	0	0	0	0	0	0	0
10	0	0.039	0.067	0.091	0.098	0.101	0.098	0.091	0.067	0.039	0
20	0	0.083	0.138	0.188	0.212	0.224	0.212	0.186	0.138	0.083	0
30	0	0.122	0.220	0.287	0.319	0.331	0.319	0.287	0.220	0.122	0
40	0	0.161	0.315	0.405	0.471	0.480	0.471	0.405	0.315	0.161	0
50	0	0.252	0.470	0.582	0.680	0.705	0.680	0.582	0.470	0.252	0
(d) ANALYTICAL RESULTS, R_z											
0	0	0	0	0	0	0	0	0	0	0	0
10	0	0.039	0.070	0.092	0.105	0.109	0.105	0.092	0.070	0.039	0
20	0	0.081	0.144	0.109	0.217	0.227	0.217	0.190	0.144	0.081	0
30	0	0.129	0.230	0.301	0.345	0.359	0.345	0.301	0.230	0.129	0
40	0	0.188	0.334	0.439	0.502	0.523	0.502	0.439	0.334	0.188	0
50	0	0.266	0.474	0.623	0.713	0.743	0.713	0.623	0.474	0.266	0

The introduced deflection was 2.4 in. with a moment arm of 11.2 in. The scale of the model was 1 in. = 5 ft. An example of the conversion of difference values, for 1R, is: $M_x = \frac{1.20}{2.54} \times \frac{1}{2.4} \times 11.8 \times 3 = 11.0$ ft-lb. The influence ordinates along line 4U are given in Col. 8, Table 1.

Moment M_y was determined in the same manner as M_x . For moment applications that did not strain the model, no appreciable deflections could be

SKUEW ANGLE

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TABLE 3.—COMPARISON OF MODEL AND ANALYTICAL RESULTS
FOR MOMENT, M , AT RIGHT ABUTMENT
(Roadway Width, 20 Ft)

Trans-verse section	INFLUENCE ORDINATES AT GRID POINTS OF SLAB										
	Left	1L	2L	3L	4L	Center	4R	3R	2R	1R	Right
(a) MODEL RESULTS; SKEW ANGLE, 0°											
4U.....	0	-0.55	-1.30	-2.28	-3.50	-5.04	-5.50	-7.50	-8.70	-9.40	-10.00
3U.....	0	-0.46	-0.94	-1.64	-2.55	-3.94	-4.76	-5.94	-6.80	-7.50	-7.50
2U.....	0	-0.28	-0.63	-1.00	-1.64	-2.55	-3.12	-4.03	-4.49	-5.04	-5.00
1U.....	0	0.18	-0.32	-0.52	-0.76	-1.10	-1.74	-2.11	-2.24	-2.50	-2.50
Center.....	0	0	0	0	0	0	0	0	0	0	0
1L.....	0	+0.18	+0.32	+0.52	+0.76	+1.10	+1.74	+2.11	+2.29	+2.50	+2.50
2L.....	0	+0.28	+0.63	+1.00	+1.64	+2.55	+3.12	+4.03	+4.49	+5.04	+5.00
3L.....	0	+0.46	+0.94	+1.64	+2.55	+3.94	+4.76	+5.94	+6.80	+7.50	+7.50
4L.....	0	+0.55	+1.30	+2.28	+3.50	+5.04	+6.50	+7.50	+8.70	+9.40	+10.00
(b) ANALYTICAL RESULTS; SKEW ANGLE, 0°											
Center.....	0	0	0	0	0	0	0	0	0	0	0
(c) MODEL RESULTS; SKEW ANGLE, 10°											
4U.....	0	-0.73	-1.83	-3.12	-4.31	-5.77	-6.77	-7.70	-8.80	-9.70	-10.13
3U.....	0	-0.64	-1.56	-2.57	-3.57	-4.49	-5.32	-6.14	-6.87	-7.60	-7.60
2U.....	0	-0.55	-1.19	-1.74	-2.75	-3.21	-3.84	-4.49	-4.85	-5.32	-5.04
1U.....	0	-0.27	-0.73	-1.10	-1.56	-1.92	-2.20	-2.47	-2.66	-2.74	-2.56
Center.....	0	-0.18	-0.36	-0.55	-0.64	-0.54	-0.54	-0.55	-0.27	-0.27	0
1L.....	0	0	+0.09	+0.37	+0.37	+0.82	+1.10	+1.37	+1.74	+2.20	+2.56
2L.....	0	+0.09	+0.55	+1.01	+1.47	+2.11	+2.75	+3.30	+3.84	+4.58	+5.04
3L.....	0	+0.55	+1.10	+1.93	+2.75	+3.57	+4.58	+5.32	+6.23	+7.15	+7.60
4L.....	0	+0.73	+1.65	+2.66	+3.85	+4.86	+5.86	+7.05	+8.43	+9.45	+10.13
(d) ANALYTICAL RESULTS; SKEW ANGLE, 10°											
Center.....	0	-0.011	-0.019	-0.031	-0.030	-0.028	-0.030	-0.031	-0.019	-0.011	0
(e) MODEL RESULTS; SKEW ANGLE, 20°											
4U.....	0	-0.46	-1.83	-3.21	-4.58	-6.05	-7.51	-8.52	-9.43	-10.50	-10.65
3U.....	0	-0.37	-1.56	-2.84	-3.94	-5.13	-6.14	-7.05	-7.60	-7.98	-7.98
2U.....	0	-0.37	-1.28	-2.20	-3.04	-3.58	-4.58	-5.22	-5.68	-5.58	-5.32
1U.....	0	-0.28	-0.83	-1.47	-2.02	-2.38	-2.93	-3.20	-3.44	-3.30	-2.66
Center.....	0	-0.09	-0.28	-0.64	-1.10	-1.02	-1.10	-0.64	-0.28	-0.28	0
1L.....	0	+0.09	0	+0.09	+0.18	+0.27	+0.55	+0.82	+1.28	+1.83	+2.66
2L.....	0	+0.45	+0.46	+0.82	+1.12	+1.56	+2.20	+2.84	+3.68	+4.30	+5.32
3L.....	0	+0.73	+0.91	+1.74	+2.47	+3.20	+4.04	+5.10	+6.14	+7.24	+7.98
4L.....	0	+1.74	+2.66	+3.67	+3.76	+5.95	+7.05	+8.42	+9.90	+10.65
(f) ANALYTICAL RESULTS; SKEW ANGLE, 20°											
Center.....	0	-0.025	-0.047	-0.067	-0.063	-0.059	-0.063	-0.067	-0.047	-0.025	0
(g) MODEL RESULTS; SKEW ANGLE, 30°											
4U.....	0	-0.64	-2.25	-3.67	-5.68	-5.96	-8.07	-9.16	-10.20	-11.00	-11.52
3U.....	0	-0.64	-1.93	-3.57	-4.76	-5.68	-6.78	-7.33	-8.07	-8.52	-8.64
2U.....	0	-0.46	-1.55	-2.94	-3.85	-4.30	-4.86	-5.40	-5.32	-5.50	-5.76
1U.....	0	-0.27	-1.19	-2.20	-2.84	-2.93	-3.20	-3.30	-3.30	-3.12	-2.88
Center.....	0	-0.18	-0.64	-0.83	-1.55	-1.37	-1.55	-0.83	-0.64	-0.18	0
1L.....	0	-0.18	-0.27	-0.27	-0.27	0	+0.46	+1.19	+1.56	+2.38	+2.88
2L.....	0	-0.18	+0.09	+0.46	+1.00	+1.74	+2.56	+3.48	+4.40	+5.50	+5.76
3L.....	0	0	+0.82	+1.28	+2.28	+3.30	+4.40	+5.68	+6.96	+8.33	+8.64
4L.....	0	+0.27	+1.28	+8.20	+3.48	+4.86	+6.24	+7.88	+9.25	+11.10	+11.52

TABLE 3.—(Continued)

Transverse section	INFLUENCE ORDINATES AT GRID POINTS OF SLAB										
	Left	1L	2L	3L	4L	Center	4R	3R	2R	1R	Right
(h) ANALYTICAL RESULTS; SKEW ANGLE, 30°											
4U.....	0	-1.32	-2.66	-3.96	-5.23	-6.47	-7.79	-9.07	-10.32	-11.55
Center.....	0	-0.036	-0.072	-0.113	-0.099	-0.091	-0.099	-0.113	-0.072	-0.036	0
4L.....	0	-0.05	+1.17	+2.43	+3.71	+5.03	+5.26	+7.54	+8.54	+10.17
(i) MODEL RESULTS; SKEW ANGLE, 40°											
4U.....	0	-1.11	-3.58	-5.13	-6.86	-8.42	-9.52	-10.72	-11.44	-12.30	-13.05
3U.....	0	-1.00	-3.30	-4.76	-6.14	-7.15	-7.95	-8.70	-8.96	-9.61	-9.89
2U.....	0	-1.00	-2.94	-4.03	-4.95	-5.50	-6.31	-6.50	-6.50	-6.67	-6.60
1U.....	0	-1.00	-2.65	-3.20	-3.93	-3.93	-4.30	-4.30	-4.10	-3.48	-3.94
Center.....	0	-1.00	-2.00	-2.28	-2.65	-2.38	-2.65	-2.19	-2.10	-1.19	0
1L.....	0	-0.91	-1.28	-1.37	-1.19	-0.64	-0.27	+0.82	+1.74	+2.75	+3.94
2L.....	0	-0.46	-0.55	-0.18	+0.46	+1.28	+2.29	+3.48	+4.76	+6.14	+6.60
3L.....	0	-0.09	+0.36	+0.64	+2.20	+3.12	+4.49	+5.77	+7.79	+9.43	+9.89
4L.....	0	+0.46	+1.00	+2.20	+3.30	+5.04	+6.41	+8.34	+10.35	+12.35	+13.05
(j) ANALYTICAL RESULTS; SKEW ANGLE, 40°											
Center.....	0	-0.060	-0.115	-0.139	-0.135	0.134	-0.135	-0.139	-0.115	-0.060	0
(k) MODEL RESULTS; SKEW ANGLE, 50°											
4U.....	0	-1.30	-3.32	-5.24	-7.21	-8.80	-10.00	-11.48	-12.61	-13.87	-15.59
3U.....	0	-1.18	-3.20	-5.12	-7.10	-8.10	-9.20	-10.46	-11.00	-11.20	-11.70
2U.....	0	-1.10	-3.04	-5.00	-7.00	-7.50	-8.00	-8.30	-8.30	-8.30	-7.79
1U.....	0	-1.00	-2.76	-4.04	-6.20	-6.50	-6.00	-6.00	-5.20	-4.36	-3.90
Center.....	0	-1.00	-2.24	-3.26	-4.32	-4.64	-4.32	-3.26	-2.24	-1.00	0
1L.....	0	-0.76	-1.56	-2.55	-3.00	-3.00	-2.25	-1.00	+1.00	+3.21	+3.90
2L.....	0	-0.64	-0.93	-1.17	-1.00	-0.64	+1.34	+3.21	+5.70	+6.80	+7.79
3L.....	0	-0.42	-0.09	+0.09	+1.00	+2.42	+3.94	+7.00	+9.84	+11.05	+11.70
4L.....	0	+0.09	+0.64	+1.35	+2.84	+4.92	+7.40	+10.81	+12.88	+15.15	+15.59
(l) ANALYTICAL RESULTS; SKEW ANGLE, 50°											
Center.....	0	-0.079	-0.164	-0.205	-0.196	-0.189	-0.196	-0.205	-0.164	-0.079	0

detected in the slab. For the model slab, the function M_y was thus taken as zero.

Test Results.—The basic frame tested had a span length of 100 ft constructed to a scale of 1 in. 5 ft. Roadway widths of 20 ft, 30 ft, and 40 ft were considered. The angle of skew was varied from 0° to 50° in 10° increments. The results of this test program are reported in Tables 2, 3, and 4, and influence contour lines are drawn for the function M_x since this is the only function that gives variable influence ordinates along transverse sections of the bridge slab. Analytical results are given first in each case so that comparisons may be made with the model results in alternate lists.

The model results for R_x and R_z reported in Table 2 are those determined for the 20-ft roadway. Results for the 30-ft and 40-ft roadways were essentially the same.

The results for M_x reported in Table 3 are for 20-ft roadway. The results for the 30-ft and 40-ft roadways are similar about the center line of the struc-

ture. The analytical comparison was made for the centerline influence ordinates only (Table 4), except in the case of the 30° skewed frame. Since it was desirable to check off center ordinates also, these calculations, which are quite laborious, were made for this frame and are reported in Table 4.

TABLE 4.—ANALYTICAL RESULTS FOR ABUTMENT MOMENTS, M_y
(These Results Are Applicable to 20-Ft, 30-Ft, and 40-Ft Roadways)

Skew angle	INFLUENCE POINTS ALONG CENTER LINE OF SLAB										
	Left	1L	2L	3L	4L	Center	4R	3R	2R	1R	Right
0	0	0	0	0	0	0	0	0	0	0	0
10	0	+0.002	+0.003	+0.008	-0.001	-0.011	-0.001	+0.008	+0.003	+0.002	0
20	0	+0.003	+0.010	+0.022	+0.003	-0.015	+0.003	+0.022	+0.010	+0.003	0
30	0	+0.005	+0.017	+0.038	+0.008	-0.025	+0.008	+0.038	+0.017	+0.005	0
40	0	+0.009	+0.041	+0.0005	-0.018	-0.039	-0.018	+0.0005	+0.041	+0.009	0
50	0	+0.016	+0.055	+0.008	+0.029	-0.063	-0.029	+0.008	+0.055	+0.016	0

VARIATION OF REACTIONS WITH SKEW ANGLE

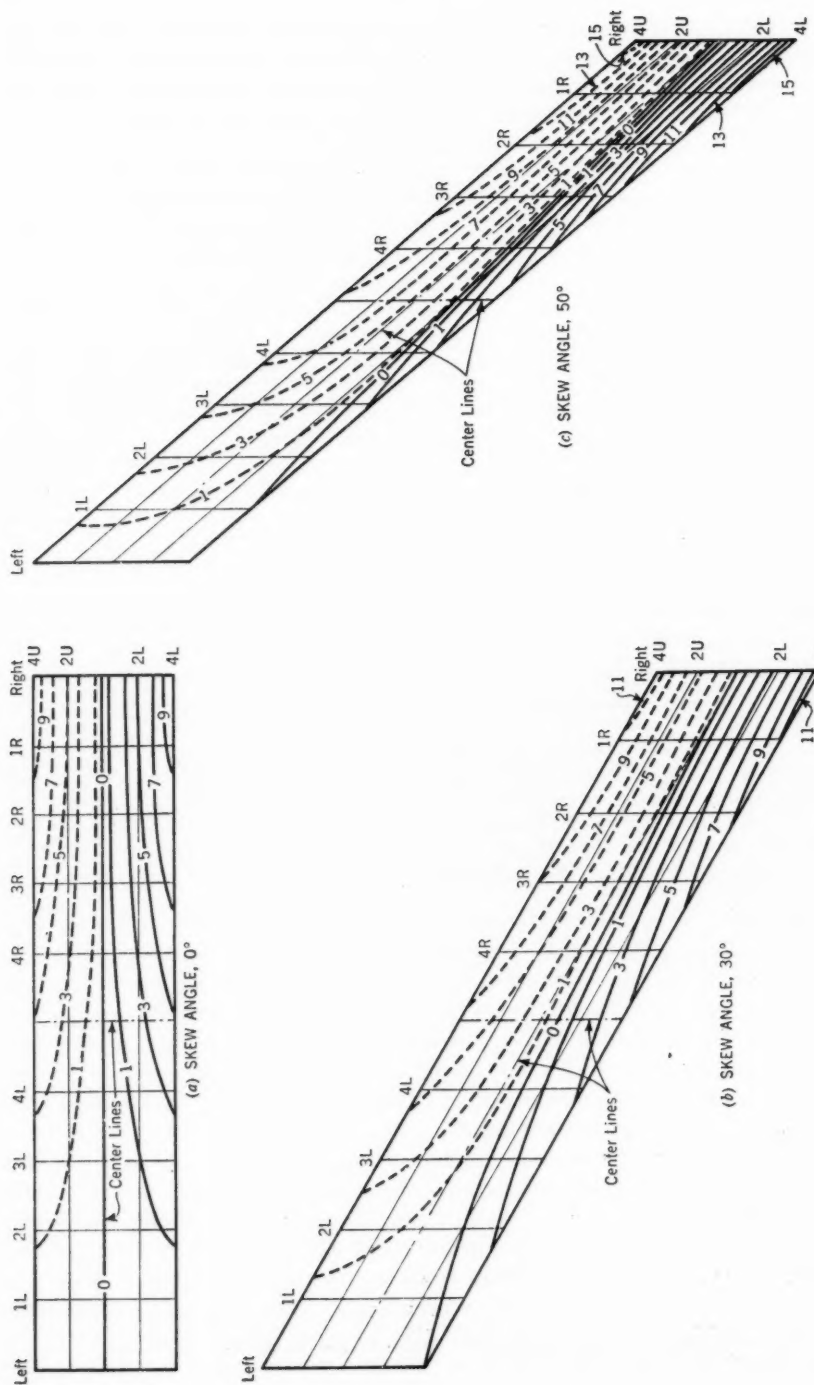
The variation of rigid-frame reactions with skew angles as determined by theory and model study can now be determined. Tables 2(a) and 2(b) indicate that the horizontal reaction R_x is essentially the same for various skew angles, as is substantiated by both the equation study (Table 2(b)) and the model study (Table 2(a)). This condition has been previously stated by Richard M. Hodges and is embodied in his important simplification of the two-hinged rigid-frame bridge. He states:⁷

“* * * The redundant reaction components H acting at the supports of a two-hinged rectangular rigid frame are so nearly identical with the corresponding redundant reaction components R_x acting at the supports of a two-hinged skewed rigid frame of the same right span, that for purposes of practical design, the difference is negligible. This fact has been attested by comparative analyses, regardless of skew, span, or deck curvature.”

The variation of the reaction component, R_z , is indicated in Tables 2(c) and 2(d). For the flat-top bridge tested, the major part of R_z is equal to $R_x \tan \epsilon$, in which ϵ is the skew angle. The reactive moment M_x is not a part of R_x and the reactive moment M_y is practically negligible in its influence. Consequently, the reaction R_x varies almost directly with the R_x -ordinates multiplied by the tangent of the skew angle. This fact has been shown by the analytical solution and substantiated by the model study. Comparison shows that results do not vary more than 5%.

The variation of the reactive moment M_x is indicated in Table 3. It can be noted from Fig. 5 that the contour diagrams are warped surfaces. The moment M_x produced the most significant disagreement between analytical and model results. Along grid lines 4U and 4L the model results and the analytical results check well within the limits of model analyses. This fact was ascertained for the frame with a skew angle of 30° (Fig. 5(b)). However,

⁷ “Simplified Analysis of Skewed Reinforced Concrete Frames and Arches,” by Richard M. Hodges, *Transactions, ASCE*, Vol. 109, 1944, p. 915.

FIG. 5.—MOMENT INFLUENCE CONTOURS FOR FUNCTION M_x AT THE RIGHT

the center-line influence ordinates show the greatest variation between model and analytical results. Since the center-line influence ordinates are normally used in design, theory will tend to give results substantially below model results. This deviation becomes pronounced as the skew angle becomes large. In the writer's opinion, the factor that contributes most to the deviation of results is the torsional factor. The value used in computations was found for beams having width-to-depth ratios of 1 to 4, whereas rigid-frame proportions may vary as much as 20 to 1.

The variation of the reactive moment M_y was found to be almost negligible for the frame tested. The model surfaces did not produce enough deflection to make deflection readings practicable.

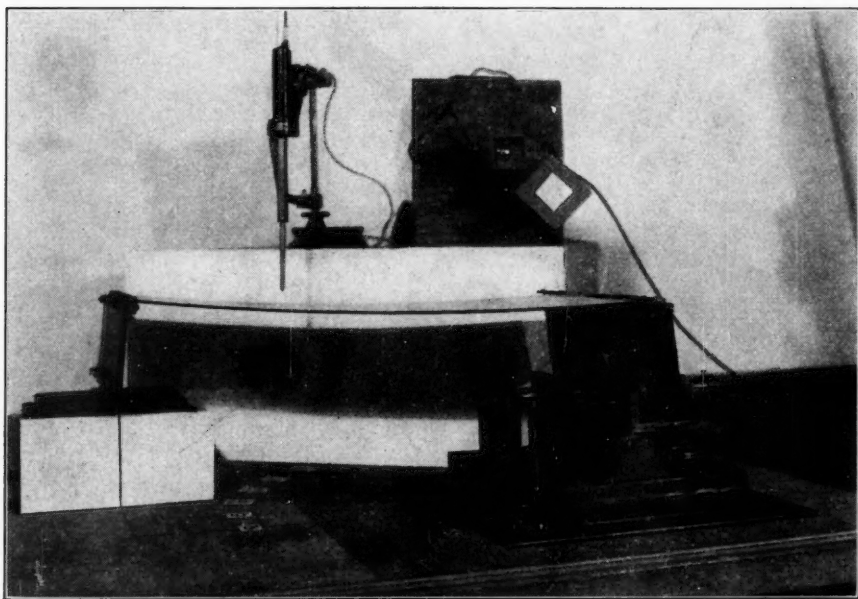


FIG. 6.—VIEW OF MODEL DEFLECTED IN THE x -DIRECTION

The test results indicate that for the two-hinged rigid-frame bridge the width of roadway does not influence the reactive elements, R_x and R_z . This fact is readily apparent when one considers that the deflected structure takes an unwarped, bowed shape for deflections along R_x and R_z . The model frame for reaction R_x is shown in Fig. 6. The model is deflected in the x -direction. The unwarped, bowed shape of the slab is typical of the results found for functions R_x and R_z . The magnitude of reactive element M_x varies almost directly with the roadway width and skew angle.

The model analysis method has proved to be very fruitful in checking the reactive elements in a skewed frame. In addition to yielding quantitative results, it has clearly indicated the qualitative trend of reactive elements.

Thus, Mr. Hodges' simplification may well be applied to such problems as the rigid frame with variable leg lengths.

ACKNOWLEDGMENT

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